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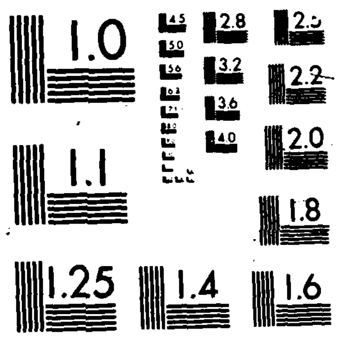
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Interim Report

ON THE PERFORMANCE OF A CLASS OF RANDOM ACCESS ALGORITHMS IN THE PRESENCE OF LIMITATIONS ON WAITING TIMES

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COMMUNICATIONS SYSTEMS LABORATORY
DEPARTMENT OF ELECTRICAL ENGINEERING
SCHOOL OF ENGINEERING AND APPLIED SCIENCES
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Abstract

~~We consider~~ the random access problem in the presence of hard limitations on the per packet waiting and access time. We describe and analyze a class of random access algorithms in this case, where the limit Poisson user model is adopted. For two specific algorithms in the class, we present quantitative results regarding output rate, delays, and proportion of rejected packets.

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1. Introduction

~~We consider~~ a packet network with independent and identical users and a common transmission channel. We require that the channel time be slotted, and that transmissions be then synchronous (each packet transmission may only start at the beginning of some slot). At the end of each slot a feedback is received. The feedback is common to all users, and contains information about the activity of the channel in the current slot.

It is assumed that if more than one packets are simultaneously transmitted during the same slot, a collision event occurs, and that the information in the transmitted packets is lost. A prespecified algorithm, performed independently by each user, is used to schedule the retransmission of collided packets in future slots. The feedback information provided by the channel is basic for the operation of the algorithm. The additional main assumption here is the existence of a limit on the waiting times per user. This limitation may be imposed by the network hardware, or may represent user impatience. An important performance measure in this case is the proportion of users that transmit their packets successfully. An additional performance measure is the delay of the successfully transmitted packets.

In this paper, we describe a class of Random Access Algorithms (RAA's), when a specific time limitation is imposed.

A method for the analysis of those algorithms is presented. The method is used to provide numerical results for specific algorithms in the class.

2. The Class of Algorithms

The algorithms require full feedback sensing in their operation; that is, each user knows the overall channel history at all times. Let us assume that at some time instant t, the packets that were generated within the interval $(0,t_1]$; $t_1<t$, have been either successfully transmitted or denied service, and there is no information concerning the interval $(t_1,t]$. In this case, the interval $(0,t_1]$ is called a "resolved interval". Then there is a constant B, such that the RAA allows all the arrivals in the interval $(t_2,t_3]$ to transmit in slot t, where $t_2 = \max(t-B,t_1)$ and $t_2 < t_3 \leq t$ (see Figure 1). The arrivals in the interval $(t_1,t_2]$ are aborted. The interval $(t_2,t]$ is called the "lag at t". The algorithms in the class are such that if there are 0 or 1 packets in the examined interval, then one slot is needed for its resolution. If the interval $(t_2,t_3]$ contains more than one packets, then a collision occurs in slot t; its resolution starts with slot t+1, and only arrivals in $(t_2,t_3]$ are allowed transmission during this collision resolution interval. Using the feedback information, each user in the system can determine the specific instant t', when all the initially collided packets are successfully transmitted. There exists, however, a constant C, such that packets that were involved in the initial collision and have not been successfully

transmitted within C slots, are aborted; a collision resolution interval has thus maximum length equal to C slots. The parameters B and C represent time constraints. B represents a limit on the waiting per packet time, and C represents a limit on the access per packet time. $B=\infty$ and $C=\infty$ represent the absence of such time constraints.

The class of algorithms we consider includes both nondynamic and dynamic RAA's. Specifically, if $t_3=t$ (i.e. the whole unexamined interval is transmitted), the algorithm is called nondynamic. If on the other hand the length of the transmitted interval is not allowed to exceed a given number Δ, the algorithm is called dynamic. The algorithms in the class are also synchronous (slotted channel).

3. User Model

The method of analysis presented in section 4 applies to the following user models:

1. The overall number of packet arrivals per slot is generated by an i.i.d. process.
2. The number of users in the system is finite, they are identical and independent, and the packet generating process per user is i.i.d., with mean λ packets/slot. Each user possesses a buffer, where he stores his nontransmitted packets on the first-come first-serve basis. The earliest stored arrival lies on the head of the buffer queue, and is called the head packet in the queue.

An interesting case of model 1 corresponds to exponential i.i.d. interarrival packet times. Model 2 has been also considered in [2], where in the absence of waiting and access time constraints, throughput-one random-access algorithms are then proposed and analyzed.

4. Analysis

Consider one of the algorithms in the class (given some RAA as in section 2). Let the system start operating at time zero, and let us consider the sequence (in time) of lags that are generally induced by the algorithm. Let C_i denote the length of the i-th lag, where $i \geq 1$. Then, the first lag corresponds to the empty slot zero; thus, $C_1=1$. In addition, the sequence $\{C_i \; ; \; 1 \leq i < \infty\}$ is a countable *Markov chain*. Let $\mathcal{D}_n$ denote the delay experienced by the n-th successfully transmitted packet arrival, as induced by the algorithm; that is, the time between the arrival of the packet and its successful transmission. Let the sequence $\{T_i\}_{i \geq 1}$ be defined as follows: Each T_i corresponds to the beginning of some slot, and $T_1=1$. In addition, each T_i corresponds to the ending point of a length-one lag. T_{i+1} is then the ending point of the first after T_i unity length lag.

Let R_i, $i \geq 0$ and F_i, $i \geq 0$ denote respectively the number of successfully transmitted packets and the number of aborted packets in the time interval $(0, T_{i+1}]$. Then, $A_i \triangleq R_i - R_{i-1}$, $i \geq 1$ and $G_i \triangleq F_i - F_{i-1}$, $i \geq 1$ denote respectively the number of successfully transmitted and the number of aborted packets in the interval $(T_i, T_{i+1}]$, where $R_0 = F_0 = 0$. The sequences $\{A_i\}_{i \geq 1}$

and $\{G_i\}_{i\geq 1}$ are clearly sequences of i.i.d. random variables; thus $\{R_i\}_{i\geq 0}$ and $\{F_i\}_{i\geq 0}$ are renewal processes. In addition, the delay process $\{\mathcal{D}_n\}_{n\geq 1}$ induced by the algorithm is regenerative with respect to the process $\{R_i\}_{i\geq 0}$, and the distribution of A_i is nonperiodic.

Let us define,

$$A \triangleq E\{A_1\} \ , \ W \triangleq E\{\sum_{i=1}^{A_1} \mathcal{D}_i\} \ , \quad H = E\{T_2 - T_1\} \tag{1}$$

It can be seen that the Markov chain $\{C_i\}$ is ergodic. In fact, for either nondynamic or dynamic with rational Δ RAA's, $\{C_i\}$ is a finite Markov chain. From the regenerative arguments [3], it follows that the rate, ρ, by which packets are successfully transmitted, and the expected steady-state delay, D, per successfully transmitted packet are respectively given by the following expressions:

$$\rho = AH^{-1} \tag{2}$$

$$D = WA^{-1} \tag{3}$$

Towards the computation of the expected values A, H, and W, let us consider some algorithm in the class, and let us then define the following quantities (see Figure 1), where t_2 and t_3 and t are as in section 2:

n_d : Number of packet arrivals in $[t_2, t_3)$, that are successfully transmitted (not aborted) during the collision resolution process, given that $t_3 - t_2 = d$.

z_d : Sum of the delays of the n_d packets, after time t.

ψ_d : Sum of the delays of the n_d packets, until the instant t_3.

ℓ_d : The number of slots needed to examine an interval of length d. Note that $d \leq C$.

$E\{X|u\}$: Conditional expectation of the random variable X, given that the length of the initially transmitted interval is u.

h_d : The number of slots needed to return to lag equal to one, when starting from a collision resolution instant with lag d.

w_d : The cumulative delay experienced by all the packets that were successfully transmitted (not aborted) during the h_d slots.

α_d : The number of packets that are successfully transmitted within the interval that corresponds to h_d.

$P(\ell|d)$: Given that the interval to be examined has length d, the probability that the corresponding collision resolution interval has length ℓ.

$$
\begin{aligned}
H_d &= E\{h_d\} \\
W_d &= E\{w_d\} \qquad (4) \\
A_d &= E\{\alpha_d\}
\end{aligned}
$$

We note that the quantities in (2), (3), and (4) are such that, $A=A_1$, $H=H_1$, and $W=W_1$. Denoting by x_d either one of the random variables h_d, w_d, α_d, the operations of any RAA in the class induce the following relationships.

$$x_d = \begin{cases} \theta_d + x_{\min(\ell,B)} & ; \quad \ell>1 \\ \theta_d & \quad \ell=1 \end{cases} \quad ; \ d \leq \Delta \tag{5}$$

$$x_d = \theta_d + x_{\min(d-\Delta+\ell,B)} \quad ; \ d>\Delta$$

; where for nondynamic algorithms, $\Delta=\infty$, and where,

$$\theta_d = \begin{cases} \ell_{\min(\Delta,d)} & ; \quad \text{for the r.v. } h_d \\ \psi_{\min(\Delta,d)} + z_{\min(\Delta,d)} + \max(d-\Delta,0)\,n_\Delta & ; \\ \qquad \text{for the r.v. } w_d & \\ n_{\min(\Delta,d)} & ; \quad \text{for the r.v. } \alpha_d \end{cases} \tag{6}$$

Taking expectations in (5), and denoting $X_d = E\{x_d\}$, we obtain:

$$X_d = E\{\theta_d\} + \sum_{\ell=2}^{C} X_{\min(\ell,B)} \ P(\ell|d) \quad ; \quad d \leq \Delta$$
$$\tag{7}$$
$$X_d = E\{\theta_d\} + \sum_{\ell=1}^{C} X_{\min(d-\Delta+\ell,B)} \ P(\ell|\Delta) \quad ; \ \Delta < d \leq B$$

; where $1 \leq d \leq B$, and where d takes at most denumerable values in $[1,B]$. Given some specific algorithm in the class, the quantities $E\{\theta_d\}$ and $P(\ell|d)$ can be computed. Those computations can be cumbersome, however, especially for finite C values.

Note that if Δ is rational, then the system in (7) is a finite system. Moreover, if Δ is an integer, then d can take only integer values in [1,B].

5. Performance Evaluation of Specific Algorithms in the Class

In this section we study the performance characteristics of specific algorithms, which belong to the class described in section 2. In particular the RAA's used are the Capetanakis protocol (CCRA) and the Massey-Capetanakis protocol (MCCRA), both in their dynamic form. We considered the operation of those algorithms in the presence of a Poisson input process, with paramter λ.

In the computations, we assumed $C=\infty$. This is mainly done for computational convenience. However, the choice $C=\infty$ approximates quite accurately the case where C is large enough and the initial arrival interval Δ is not very long. In tables 1,2,3,4, and 5, we include expected delays of the successfully transmitted packets as well as the output traffic rate and the proportion of rejected packets, for various values of the parameters λ, Δ, and B.

Given constraints on the proportion of rejected packets and the maximum expected delay of the successfully transmitted packets, it is of interest to select the parameters Δ and B in a way that the input traffic rate is maximized. Specifically, given constants e_1, e_2, and the parameters Δ and B, we define,

$$\lambda^*_{e_1,e_2}(\Delta,B) = \sup(\lambda \; : \rho \le e_1 \; , \; D \le e_2) \tag{8}$$

Given e_1, e_2, we then wish to find the rate:

$$\lambda^{*}_{e_1,e_2} = \sup_{\Delta,B} \lambda^{*}_{e_1,e_2}(\Delta,B) \tag{9}$$

Given e_1 and e_2, and either the CCRA or the MCCRA protocols, approximate values of $\lambda^{*}_{e_1,e_2}$ can be extracted from tables 1 to 5. For example, for the CCRA and $e_1=0.05$, $e_2=4$, an approximate optimal (Δ,B) choice corresponds to $\Delta=2$ and $B=5$. Then, $\lambda^{*}_{0.05,4} \simeq \lambda^{*}_{0.05,4}(2,5) = 0.31$, and $D=3.2$. For the CCRA and $e_1=0.1$, $e_2=20$, the approximate optimal (Δ,B) choice corresponds to $\Delta=2$ and $B=20$, which give, $\lambda^{*}_{0.1,20} \simeq \lambda^{*}_{0.1,20}(2,20)=0.47$ and $D=19$. Similar choices can be made for the MCCRA protocol.

Next, we examine the case when C takes small values. In the computation we assumed $C=5$. In table 6 we include the average delay of the successfully transmitted packet as well as the probability of rejection for various values of the parameters λ, B.

6. Conclusion

We presented a framework for the analysis of a class of algorithms, subject to strict limitations on the waiting times. For specific requirements on the acceptable rejection rates and on the average waiting times, the analysis provides a methodology for the determination of those algorithmic parameters which maximize the acceptable input rate. The limitations on the waiting times considered in this paper mainly correspond to hardware restrictions. It is interesting to study the behavior of the algorithms when the acceptable per user waiting time is a random variable. The latter is a better representation of user impatience.

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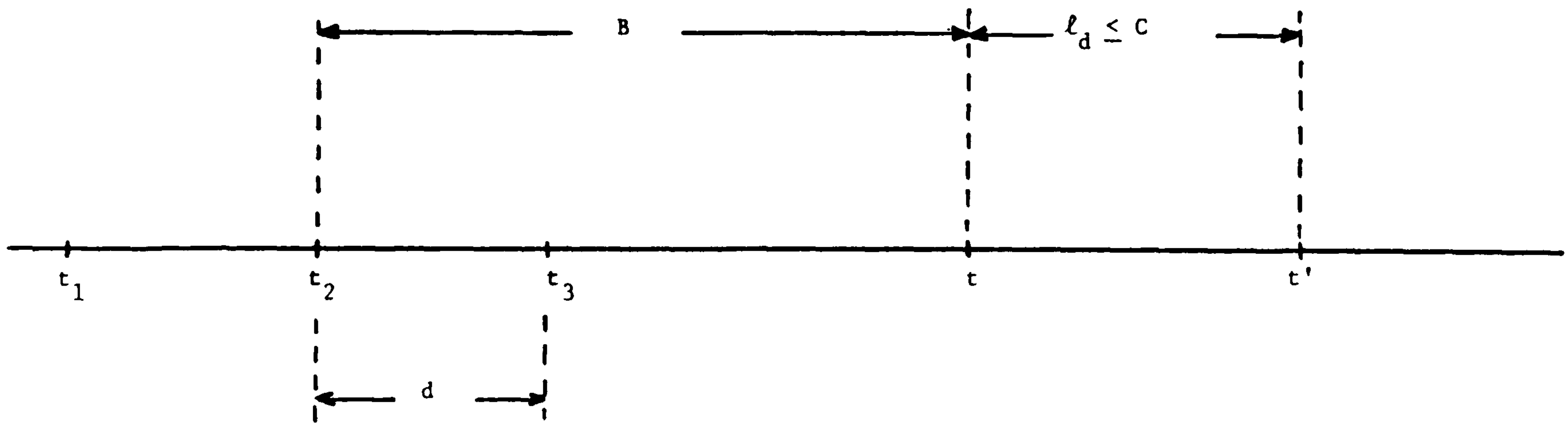

Figure 1

	B=5 , C=∞			B=10 , C=∞		
λ	mean-delay	output rate	rejected proportion of pcks	mean-delay	output rate	rejected proportion of pcks
0.0500	0.16692 E+01	0.04999	0.88833 E-04	0.17503 E+01	0.04999	0.96415 E-05
0.1000	0.18803 E+01	0.14985	0.96791 E-03	0.22067 E+01	0.09999	0.10025 E-03
0.1500	0.21377 E+01	0.14940	0.40400 E-02	0.28743 E+01	0.14989	0.69946 E-03
0.2000	0.24442 E+01	0.19776	0.11184 E-01	0.37388 E+01	0.19947	0.26372 E-02
0.3000	0.31987 E+01	0.28637	0.45418 E-01	0.58475 E+01	0.29565	0.14494 E-01
0.4000	0.40970 E+01	0.35495	0.11261 E+00	0.79567 E+01	0.37763	0.55906 E-01
0.5000	0.50507 E+01	0.39744	0.20510 E+00	0.96588 E+01	0.42011	0.15976 E+00
0.6000	0.59721 E+01	0.41728	0.30452 E+00	0.10919 E+02	0.42767	0.28720 E+00
0.7000	0.68079 E+01	0.42279	0.39600 E+00	0.11864 E+02	0.42619	0.39115 E+00

Table 1

$\Delta=2$, CCRA

	B=5 , C=∞			B=10 , C=∞		
λ	mean delay	output rate	rejected proportion of pcks	mean-delay	output rate	rejected proportion of pcks
0.0500	0.16651 E+01	0.04999	0.81127 E-04	0.17280 E+01	0.04999	0.99948 E-05
0.1000	0.18677 E+01	0.09991	0.87079 E-03	0.21313 E+01	0.09999	0.66754 E-04
0.1500	0.21183 E+01	0.14944	0.36754 E-02	0.27419 E+01	0.14991	0.61021 E-03
0.2000	0.24259 E+01	0.19792	0.10407 E-01	0.35767 E+01	0.19956	0.22012 E-02
0.3000	0.32304 E+01	0.28662	0.44599 E-01	0.57976 E+01	0.29603	0.13223 E-01
0.4000	0.42630 E+01	0.35378	0.11552 E+00	0.82112 E+01	0.37708	0.57248 E-01
0.5000	0.54222 E+01	0.39226	0.21547 E+00	0.10234 E+02	0.41546	0.16907 E+00
0.6000	0.65791 E+01	0.40680	0.32199 E+00	0.11743 E+02	0.41512	0.30812 E+00
0.7000	0.76435 E+01	0.40780	0.41742 E+00	0.12876 E+02	0.40876	0.41605 E+00

Table 2

$\Delta=3$, CCRA

$B=20$, $C=\infty$			
λ	mean delay	output rate	proportion of rejected pcks.
0.0500	0.20980 E+01	0.05000	0.00000 E+00
0.1000	0.36051 E+01	0.10000	0.00000 E+00
0.1500	0.59663 E+01	0.14999	0.87561 E-05
0.2000	0.88721 E+01	0.19999	0.95554 E-04
0.3000	0.14325 E+02	0.29947	0.17674 E-02
0.4000	0.17754 E+02	0.39134	0.21636 E-01
0.4500	0.18854 E+02	0.42253	0.61037 E-01
0.5000	0.19706 E+02	0.42738	0.14523 E+00
0.6000	0.20940 E+02	0.42943	0.28429 E+00
0.7000	0.21843 E+02	0.42783	0.39031 E+00

Table 3

$\Delta=2$, CCRA

	B=5 , C=∞			B=10 , C=∞		
λ	mean-delay	output rate	rejected proportion pcks	mean-delay	output rate	rejected proportion of pcks
0.0500	0.16359 E+01	0.04996	0.79671 E-03	0.16443 E+01	0.04999	0.89457 E-04
0.1000	0.17972 E+01	0.09966	0.33661 E-02	0.18343 E+01	0.09996	0.42339 E-03
0.1500	0.19885 E+01	0.14877	0.82036 E-02	0.20855 E+01	0.14981	0.12283 E-02
0.2000	0.22162 E+01	0.19678	0.16093 E-01	0.24206 E+01	0.19939	0.30199 E-02
0.3000	0.27938 E+01	0.28646	0.45114 E-01	0.34624 E+01	0.29566	0.14468 E-01
0.4000	0.35365 E+01	0.36093	0.97669 E-01	0.51507 E+01	0.37909	0.52260 E-01
0.5000	0.43988 E+01	0.41312	0.17375 E+00	0.72943 E+01	0.43362	0.13275 E+00
0.6000	0.52962 E+01	0.44221	0.26297 E+00	0.92843 E+01	0.45603	0.23994 E+00
0.7000	0.61471 E+01	0.45393	0.35152 E+00	0.10759 E+02	0.46009	0.34272 E+00

Table 4

$\Delta=2$, MCCRA

	B=5 , C=∞			B=10 , C=∞		
λ	mean-delay	output rate	proportion rejected pcks	mean-delay	output rate	proportion of rejected pcks
0.0500	0.16336 E+01	0.04996	0.79542 E-03	0.16401 E+01	0.04999	0.89031 E-04
0.1000	0.17895 E+01	0.0996	0.33496 E-02	0.18186 E+01	0.0996	0.41728 E-03
0.1500	0.19758 E+01	0.14877	0.81431 E-02	0.20528 E+01	0.14982	0.12017 E-02
0.2000	0.22007 E+01	0.19680	0.15978 E-01	0.23671 E+01	0.19941	0.29536 E-02
0.3000	0.27975 E+01	0.28640	0.45310 E-01	0.33698 E+01	0.29567	0.14443 E-01
0.4000	0.36164 E+01	0.35995	0.10012 E+00	0.50818 E+01	0.37845	0.53878 E-01
0.5000	0.46261 E+01	0.40930	0.18139 E+00	0.74014 E+01	0.42977	0.14046 E+00
0.6000	0.57251 E+01	0.43378	0.27702 E+00	0.96905 E+01	0.44623	0.25627 E+00
0.7000	0.67970 E+01	0.44067	0.37046 E+00	0.11459 E+02	0.44466	0.36476 E+00

Table 5

$\Delta=3$, MCCRA

C=5				
λ	B=5, Δ=2 probability of rej.	B=10, Δ=2 probability of rej.	B=5, Δ=2 average delay	B=10, Δ=2 average delay
0.0500	0.9498 E - 04	0.12417 E - 04	0.14614 E + 01	0.14617 E + 01
0.1000	0.1103 E - 02	0.30082 E - 03	0.15896 E + 01	0.15930 E + 01
0.1500	0.7712 E - 02	0.29784 E - 02	0.17502 E + 01	0.17679 E + 01
0.2000	0.1678 E - 01	0.94013 E - 02	0.19450 E + 01	0.20029 E + 01
0.3000	0.8004 E - 01	0.67089 E - 01	0.26443 E + 01	0.29685 E + 01
0.4000	0.1598 E + 00	0.11609 E + 00	0.33020 E + 01	0.43602 E + 01
0.5000	0.2800 E + 00	0.23958 E + 00	0.42174 E + 01	0.64988 E + 01
0.6000	0.3685 E + 00	0.34043 E + 00	0.50442 E + 01	0.84686 E + 01
0.7000	0.4462 E + 00	0.43606 E + 00	0.58683 E + 01	0.99069 E + 01

Table 6

CCRA

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